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ON THE VARIETIES OF FINITISM

Mohammad Saleh Zarepour

Defenders of the Kalām Cosmological Argument appeal to the so-called Hilbert’s Hotel Argument to establish the finitude of the past based on the impossibility of actual infinites. Some of their opponents argue that this proves too much because if the universe cannot be beginningless due to the impossibility of actual infinites, then, for the same reason, it cannot be endless either. Discussing four different senses of the existence of an actual infinite, I criticize both sides of the debate by showing, on the one hand, that the Hilbert’s Hotel Argument is not powerful enough to rule out the possibility of the infinitude of the past and, on the other hand, that the soundness of the argument for the finitude of the past from the impossibility of actual infinites does not establish the soundness of the parallel argument for the finitude of the future.

The Kalām Cosmological Argument (KCA) states that the universe has a cause because, on the one hand, the universe has a beginning and, on the other hand, everything that has a beginning has a cause.¹ The main argument which the proponents of KCA employ to show that the universe is not beginningless can be formulated as follows:

Argument A

(A1) The universe exists.

(A2) If the universe has no beginning, an infinite temporal regress of events exists.

(A3) An infinite temporal regress of events is an actual infinite.

(A4) An actual infinite cannot exist.

¹See, for example, Craig, The Kalām Cosmological Argument, 63, and idem., Reasonable Faith, 111.
Therefore:

\[(A5)\] The universe has a beginning.²

Some opponents of KCA have argued that if we reject the infinitude of the past based on the impossibility of actual infinites, we have no option but to reject the infinitude of the future based on the same consideration.³ This is because we can easily devise an argument parallel to Argument A but in favour of the finitude of the future. Such a parallel argument can be articulated as follows:

*Argument B*

\[(B1)\] The universe exists.

\[(B2)\] If the universe has no end, an infinite temporal progress of events exists.

\[(B3)\] An infinite temporal progress of events is an actual infinite.

\[(B4)\] An actual infinite cannot exist.

Therefore:

\[(B5)\] The universe has an end.⁴

Compared to the infinitude of the past, the infinitude of the future plays a more crucial role in religious belief; indeed it seems there is greater consensus among theists on the latter thesis than on the former. The

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²It is worth noting that this argument—which is based on the impossibility of the existence of the infinite—differs from the argument from the impossibility of traversing the infinite. For discussions on the latter argument, upon which I do not touch in this paper, see, for example, Craig, *The Kala姆 Cosmological Argument*, 39–41; *Reasonable Faith*, 122–123; and Craig and Sinclair, “Kalaム Cosmological Argument,” 118–119. Argument A is constructed by adding (A1) and (A2) to the premises of a famous argument for the finitude of the past that is discussed, among others, by Craig, *The Kalaム Cosmological Argument*, 69, *Time and Eternity*, 221, and *Reasonable Faith*, 116, and Craig and Sinclair, “Kalaム Cosmological Argument,” 103. They couple (A3) with (A4) and conclude that an infinite temporal regress of events cannot exist. They then take the latter conclusion as an equivalent for the claim that the universe cannot be beginningless. This indicates that they accept (A1) and (A2), even though they do not explicitly put them as premises in the formal structure of their argument. Although (A2) seems to be an entirely innocent premise, Puryear (in his “Finitism and the Beginning of the Universe”) has argued that it is incompatible with (A4). As a result, this line of argument in favour of the finitude of the past fails, or so Puryear concludes. Since in the present paper I am concerned with a different issue, I do not engage with Puryear’s objection, even though I am sympathetic with it. I have elsewhere discussed the strength of his objection and the scope of its impact on the various arguments for the finitude of the past. See Zarepour, “Infinite Magnitudes.”


⁴This argument is constructed by adding (B1) and (B2) to the premises of an argument for the finitude of the future that is proposed by Malpass and Morriston in “Endless and Infinite,” 831. Adopting their terminology, I use “progress” as the future-oriented dual of “regress.”
evidence for this claim is that many theists—regardless of whether or not they accept the finitude of the past—believe in the life everlasting which includes infinite progresses of heavenly or hellish events. In other words, (B5) seems to contradict the doctrine of the endless afterlife, which is a central element of many religious systems of beliefs. That is why emphasizing that Argument A and Argument B enjoy the same degree of plausibility is a powerful strategy for encouraging theists to give up the former. To protect Argument A against this line of attack, the friends of KCA usually argue that there is a dissimilarity between the past and the future because of which Argument B is not as defensible as the parallel argument against the infinitude of the past. For example, Craig contends that the future is only potentially infinite and that (B3), by contrast with (A3), is false. To describe the difference between actual and potential infinities, he says:

An actual infinite is a collection of definite and discrete members whose number is greater than any natural number 0, 1, 2, 3… This sort of infinity is used in set theory to designate sets that have an infinite number of members, such as [0, 1, 2, 3…]. The symbol for this kind of infinity is the Hebrew letter aleph: \( \aleph \). The number of members in the set of natural numbers is \( \aleph_0 \).

By contrast, a potential infinite is a collection that is increasing toward infinity as a limit but never gets there. The symbol for this kind of infinity is the lemniscate: ∞.

The asymmetry between the past and the future is explained by Craig and Sinclair as follows:

[W]hen we say that the number of past events is infinite, we mean that prior to today, \( \aleph_0 \) events have elapsed. But when we say that the number of future events is infinite, we do not mean that \( \aleph_1 \) events will elapse, for that is false. Ironically, then, it turns out that the series of future events cannot be actually infinite regardless of the infinity of the past or the metaphysical possibility of an actual infinite, for it is the objectivity of temporal becoming that makes the future potentially infinite only.

In brief, one side in this debate argues that if Argument A is sound, then so is Argument B. The other side denies this equivalence, claiming that (B3), unlike (A3), is false. So the crux of the debate seems to be whether or not an endless progress of events can be counted as an actual infinite. Aside from this, both camps apparently agree that if (B3) is true, then the impossibility of the existence of an actual infinite implies not only that the universe has a beginning but also that it has an end. One of the aims of the present paper is to challenge this consensus. I will argue that even if the truth of (B3) is guaranteed, the finitude of the past does not commit one to the finitude of the future. This is because there are multiple senses in which an actual infinite could be said to exist, and the impossibility of an actual infinite existing in one of those senses does not necessarily

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5 See Malpass and Morriston, “Endless and Infinite,” 831.
6 Craig, Reasonable Faith, 116.
prove the impossibility of an actual infinite existing in another sense. In particular, it can be shown that there is a specific sense of the existence of an actual infinite whose impossibility implies the finitude of the past but not that of the future.

Both of the above arguments rely on the premise that an actual infinite cannot exist. The advocates of KCA argue that although there is no logical inconsistency in the notion “actual infinite,” it is metaphysically (or ontologically) impossible for an actual infinite to be instantiated in the mind-independent real world. In other words, actual infinites are metaphysically impossible, though logically possible. That is why the defenders of KCA acknowledge that working with actual infinites in a purely mathematical framework which carries no ontological commitments is unproblematic and does not raise any logical inconsistency. It is only the existence or instantiation of actual infinites in the mind-independent real world that leads to absurd consequences like the equivalence of an existing collection to some of its proper subcollections. We know that the members of every actually infinite collection of things can be put in one-to-one correspondence with the members of some of its proper subcollections. This can be taken as evidence that the number of members of an actually infinite collection is equal to the number of members of some of its proper subcollections. But since the instantiation of such an equality in the mind-independent world seems to be absurd, an actually infinite collection cannot exist in the real world. It is worth noting that the equivalence between infinite collections and their proper subcollections is not assumed to be logically impossible. The claim is merely that such equivalences cannot be exemplified in the real world.

To show how absurd is the instantiation of such a collection-subcollection equivalence in the real world, the advocates of KCA usually appeal to the Hilbert’s Hotel Argument (HHA). Hilbert’s Hotel is an extraordinary hotel with an actually infinite number of similar rooms which are successively numbered with natural numbers starting from 1 (in such a way that no room is left without a number). It can be shown that the number of, for example, the odd-numbered rooms is equal to the number of all the rooms. Accordingly, if such a hotel exists in the real world, the amount of materials used to build the odd-numbered rooms would be equal to the amount of materials used to build all the rooms. For instance, if all the parts of such hotels are made of iron, then by melting the parts of only the odd-numbered rooms, a new hotel which includes all the odd-numbered rooms as well as all the even-numbered rooms can be built. But this seems to be quite implausible. Our intuitions about existing objects resist

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9See, for example, Craig and Sinclair, “Kalam Cosmological Argument,” 107.
such consequences. The upshot is that actual infinites cannot exist in the real world; otherwise, despite the absurdity of Hilbert’s Hotel, it could be instantiated in the real world.

I agree with the friends of KCA that an actually infinite collection whose members are supposed to exist all together simultaneously—e.g., Hilbert’s Hotel, whose rooms are supposed to exist all together simultaneously—cannot be instantiated in the real world. But I think it would be too hasty to generalize this observation and conclude that an actually infinite collection cannot exist in any sense. To see why, consider the following four different senses of the existence of a collection:

(a) There is a moment of time $t$ such that every member of the collection exists at $t$.

(b1) There is a moment of time $t$ such that every member of the collection exists either at $t$ or at some moment of time before $t$.

(b2) There is a moment of time $t$ such that every member of the collection exists either at $t$ or at some moment of time after $t$.

(c) Every member of the collection exists at one or more moments of time.

If a collection exists in the sense of (a), there is a moment of time at which all its members exist all together simultaneously. But if a collection exists in any of the three other senses, its different members can in principle exist at different moments of time. There might be no moment at which all the members exist simultaneously. Accordingly, the existence of a collection in the sense of (a) entails its existence in each of the senses (b1), (b2), and (c); but the existence of a collection in none of the three latter senses entails its existence in the sense of (a). If a collection exists in the sense of (b1), there is a moment of time up until which every member of the collection has existed for some time; whereas if a collection exists in the sense of (b2),

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10 For discussions of HHA see, among others, Craig, *Time and Eternity*, 222–223 and *Reasonable Faith*, 118–119, and Craig and Sinclair, “Kalam Cosmological Argument,” 108–110. Morriston (in his “Craig on the Actual Infinite”) has meticulously shown that Craig’s ground for denying the possibility of the instantiation of collection-subcollection equivalences in the real world is the absurdity of the whole-part equality.

11 *Argument A* and *Argument B* talk about, respectively, regresses and progresses of events. Craig and Sinclair (in their “Kalam Cosmological Argument,” 106) analyze events as changes and argue that “since changes take time, there are no instantaneous events so defined.” According to this view, events occur during temporal intervals, not at singular moments of time. If we accept this view, then it would be more accurate to revise our formulation of the above four senses of the existence of a collection by replacing “moment of time” with “finite temporal interval.” However, for the sake of simplicity, and since this change has no important impact on the main point I would like to make, I stick to what I have already presented above and assume that the infinitudes of the past and of the future entail the existence of, respectively, regresses and progresses of events each of which instantaneously happens. The above quadratic distinction is a generalized version of an Avicennian binary distinction between the senses (a) and (c) of the existence of a whole. See Zarepour, “Avicenna on Mathematical Infinity,” sec. 4.3 and “Infinite Magnitudes,” sec. 4.
there is a moment of time after which every member of the collection will exist for some time. The existence of a collection in the sense of (b1) does not entail its existence in the sense of (b2), nor vice versa. Nonetheless, they both entail the existence of the collection in the sense of (c). Indeed, among the four senses above, (c) expresses the weakest sense of the existence of a collection. Although (c) is implied by each of the other three, none of those three are implied by (c).

Considering the above distinctions, we can now see that if the universe has no beginning, an actually infinite temporal regress of events exists in the senses of (b1) and, consequently, (c). It does not exist in the senses of either (a) or (b2). As a result, Argument A is sound only if we can show either that an actual infinite cannot exist in the sense of (c) or, at least, that an actual infinite cannot exist in the sense of (b1). Thus, Argument A must be understood as an imprecise formulation of the following argument:

**Argument A**

(A1) The universe exists.

(A2) If the universe has no beginning, an infinite temporal regress of events exists in the sense of (b1).

(A3) An infinite temporal regress of events is an actual infinite.

(A4) An actual infinite cannot exist in the sense of (b1).

Therefore:

(A5) The universe has a beginning.

On the other hand, if the universe has no end, an actual infinite temporal progress of events exists in the senses of (b2) and, consequently, (c). It does not exist in the senses of either (a) or (b1). This indicates that Argument B is sound only if we can show either that an actual infinite cannot exist in the sense of (c) or, at least, that an actual infinite cannot exist in the sense of (b2). This means that Argument B is best understood as an imprecise formulation of the following argument:

**Argument B**

(B1) The universe exists.

(B2) If the universe has no end, an infinite temporal progress of events exists in the sense of (b2).

(B3) An infinite temporal progress of events is an actual infinite.

(B4) An actual infinite cannot exist in the sense of (b2).
Therefore:

(B5) The universe has an end.\(^{12}\)

Regarding these arguments, two important observations can be made: First, the defenders of KCA have failed to establish the soundness of Argument A* by appealing to HHA. Second, the soundness of Argument A* is logically independent from that of Argument B*. Thus, even if the former argument is sound, this does not show that the latter is sound too. To justify the first claim, I start by highlighting the fact that HHA, if sound, shows only that an actual infinite cannot exist in the sense of (a). All the rooms of Hilbert’s Hotel are supposed to exist all together simultaneously. But the impossibility of the existence of a collection in the sense of (a) does not establish the impossibility of its existence in either of the three other senses. So, HHA on its own cannot exclude the possibility of the existence of an actual infinite in the senses other than (a). In particular, this argument cannot prove that an actual infinite cannot exist in the sense of (b1). In other words, HHA does not justify (A*4). So, the soundness of Argument A* cannot be established based on HHA.

Now suppose, for the sake of argument, that a convincing justification for the impossibility of the existence of actual infinites in the sense of (b1) is provided. If so, Argument A* would be sound. Nevertheless, this does not automatically guarantee the soundness of Argument B*. As we saw, the existence of a collection in the sense of (b1) is logically independent from its existence in the sense of (b2). Neither can be derived from the other. Therefore, the impossibility of the existence of an actually infinite collection in the sense of (b1)—which is what (A*4) asserts—does not establish the impossibility of its existence in the sense of (b2)—which is what (B*4) asserts. Therefore, the soundness of Argument A* does not establish the soundness of Argument B*.

This approach to saving the possibility of the infinitude of the future while denying the possibility of an infinite past radically differs from Craig’s. He would repudiate Argument B* because he denies that an infinite temporal progress of events forms an actual infinite—i.e., for him (B3) is false. I think, however, that Craig’s critics are right to insist that if we endorse his definition of an actual infinite (which I quoted earlier in the paper), we must describe an infinite temporal progress of events as an actual infinite.\(^{13}\) Such a progress is composed of a collection of distinct events which will happen in the future and whose number is greater than

\(^{12}\)If, in Argument A* and Argument B*, “(b1)” and “(b2)” are substituted with “(c),” the resulting arguments would still be valid but, compared to the original arguments, less likely to be sound. This is because the impossibility of the existence of actual infinites in the sense of (c) represents the most general sense of the impossibility of the existence of actual infinites and is the most difficult claim of this type to defend.

\(^{13}\)See, among others, Morriston, “Beginningless Past,” and Malpass and Morriston, “Endless and Infinite.”
any natural number. Admittedly, the number of the events that will have happened as time passes is increasing toward infinity and must be counted, therefore, as only potentially infinite. Nevertheless, the number of the events that will happen in the future is not changing; it is actually infinite. An infinite temporal progress of events satisfies the conditions of being an actual infinite.\textsuperscript{14} However, this alone does not enable Craig’s critics to establish the finitude of the future based on the finitude of the past. My discussion shows that a more carefully calibrated reformulation of Craig’s argument for the finitude of the past (from the impossibility of actual infinites) cannot be easily transformed into an argument for the finitude of the future. This is because the possibility of the existence of an infinite temporal regress of events can be ruled out just based on the impossibility of the existence of an actual infinite in the sense of (b1); but this impossibility is not strong enough to rule out the possibility of the existence of an infinite temporal progress of events whose existence would obviously be in the sense of (b2).

One might complain that HHA shows that actual infinites cannot exist in any of the four aforementioned senses because this argument shows that every actual infinite is equivalent to some of its proper subcollections. If so, HHA would justify both (A*4) and (B*4) and my objections to Craig (i.e., that Argument A* is not sound) and to his opponents (i.e., that the soundness of Argument A* does not establish that of Argument B*) are untenable. However, I think that this complaint is ill-founded. The members of an actually infinite collection—regardless of whether or not it exists in any of the four aforementioned senses—can be put in one-to-one correspondence with the members of some of its proper subcollections. Thus, every actually infinite collection is equivalent to some of its proper subcollections; but such equivalences, as Craig affirms, are not logically absurd. That is why working with actual infinites in an ontologically neutral framework (e.g., a purely formalistic version of the Cantorian set theory) does not necessarily raise any contradiction. But if “no collection is equivalent to its subcollection” does not express a logical truth, then it might also be possible for a collection that is equivalent to some of its proper subcollections to exist in some of the aforementioned senses of the existence of a collection. The possibility of the existence of an actually infinite collection in the sense of (x) cannot be excluded just by appealing to the fact that an actually infinite collection is equivalent to some of its proper subcollections.\textsuperscript{15} Rather, we also need to say something on why a collection-subcollection equivalence

\textsuperscript{14}Indeed, the core of the argument proposed by Malpass and Morriston (in their “Endless and Infinite”) can be summarized like this: Even if the future is endless, the number of the events that will have happened as time passes is only potentially infinite. Nevertheless, if the future is endless, then the number of the events that will happen is actually infinite because the set of such events can be put in one-to-one correspondence with the set of natural numbers. This indicates that the set of the events that will happen satisfies Craig’s definition of an actually infinite set. Therefore, if there can be no actual infinite, the number of the events that will happen cannot be infinite either. So the future cannot be endless.

\textsuperscript{15}Hereafter “(x)” can be replaced with any of “(a),” “(b1),” “(b2),” and “(c).”
is intolerable if the collection exists in the sense of (\(\chi\)). The existence of Hilbert’s Hotel cannot be rejected just based on the fact that the number of its odd-numbered rooms, for example, is equal to the number of all its rooms. Rather, we need an explanation of why such an equality has counterintuitive consequences when the hotel exists in the sense of (\(a\)). Even if HHA can successfully provide such an explanation, it has nothing to say on why such an equality is absurd if the collection exists in another sense. Thus, it cannot justify either (\(A^*4\)) or (\(B^*4\)).

I showed that the soundness of Argument \(B^*\) is logically independent from that of Argument \(A^*\). Accordingly, endorsing the latter argument while refusing to endorse the former raises no logical inconsistency. This claim implies that it is not the case that every justification for the soundness of Argument \(A^*\) can automatically be transformed to a justification for the soundness of Argument \(B^*\). This is mainly because there can in principle be a justification for (\(A^*4\)) which fails to justify (\(B^*4\)). Now the critic of KCA might say that even if we accept that the truths of these two premises

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16A potential candidate for such an explanation could be something on the following lines: The equality mentioned above shows that the amount of materials needed to build the odd-numbered rooms would be equal to the amount of materials needed to build all the rooms. This sounds as if infinitely many new rooms can be added to the odd-numbered rooms without adding new materials. The odd-numbered rooms can be built first; then they can be destroyed, and the materials used in them can be used to build the whole hotel. This blatantly clashes with our intuitions regarding the real world. Therefore, such a hotel cannot exist. Admittedly, this explanation is not strong enough to rule out the possibility of the existence of all infinite collections in the sense of (\(a\)) regardless of what the nature of the members of such a collection is. For example, this explanation reveals no inconsistency in the assumption of the existence of an infinite collection of simple immaterial things (e.g., souls, if they exist). See my discussion of Avicenna’s view regarding the existence of an infinite number of souls in Zarepour, “Avicenna on Mathematical Infinity,” sec. 4.3.

17Suppose that there is a room builder who has a certain amount of materials by which she can build only one room. She builds a room every day but destroys it at the end of the day and builds another room with the same materials the next day. If the universe is beginningless and the room builder has been repeating the described process as long as time existed, then the collection of all the rooms built up until yesterday exists in the sense of (\(b_1\)). This collection is an actual infinite because if we assign the natural number \(n\) to the room that was built \(n\) days ago, then this collection would be in one-to-one correspondence with the set of natural numbers starting from 1. Here again the number of, for example, the odd-numbered rooms is equal to the number of all the rooms. However, it is not clear how this logically innocent equality might cause a metaphysically absurd consequence. For sure, the explanation we suggested in the previous footnote for the case of Hilbert’s Hotel is useless here. All the rooms have been built with the same materials; and this assumption seems to be uncontroversial. This shows that an explanation for why a collection-subcollection equivalence is unacceptable if the collection exists in the sense of (\(a\)), does not necessarily say anything about the absurdity of the same equivalence when the collection exists in another sense.

18As it is suggested by Loke (in his “No Heartbreak at Hilbert’s Hotel”), another way of trying to make the supposed impossibility of a Hilbert’s Hotel relevant to premise (\(A^*4\)) would be to argue that if a beginningless series of events were possible, then it would be possible for God to have been creating hotel rooms \textit{ex nihilo} at regular intervals throughout the beginningless past. If such were the case, then there would already a Hilbert Hotel exists. But since such a hotel cannot exist, the past cannot be beginningless. In “Infinite Magnitudes,” I have discussed and rebutted this objection. See also Hedrick, “Once More to the Hotel,” for another treatment of this objection.
are logically independent from each other, we can still insist that we have been given no reason to endorse \((A^*4)\) that is not also a reason to endorse \((B^*4)\). I of course agree with the latter claim but only because I believe that \((A^*4)\) is false and we cannot at all be given any convincing justification for it. Nevertheless, I believe that there are arguments for \((A^*4)\) which have nothing to do with \((B^*4)\). As I said, I think that such arguments cannot be sound, but their unsoundness is not due to the fact that they have the counterintuitive consequence that the future is finite. They may have no such consequence. Suppose for example that someone has proved that the number of things that have existed or events that have happened cannot actually be infinite. Such a proof might be unacceptable. But this is definitely not because it entails the finitude of the future. The finitude of the future is quite compatible with the claim that the number of the events that will have happened as time passes will never be actually infinite.

To recapitulate, there seem to be four different senses of the existence of an actual infinite and, correspondingly, four different senses of finitism. According to the finitism in the sense of \((x)\), an actual infinite cannot exist in the sense of \((x)\). To show that the universe has a beginning, we need to establish finitism in the sense of \((b1)\). However, HHA at best proves finitism in the sense of \((a)\). Since finitism in the sense of \((b1)\) does not automatically follow from finitism in the sense of \((a)\), the finitude of the past cannot be concluded from HHA. This shows that Craig’s argument that the universe has a beginning (at least when it is grounded in HHA) does not work. On the other hand, even if we accept that the universe has a beginning because finitism in the sense of \((b1)\) is true, this does not necessitate that the universe has an end. To show that the universe has an end, we need to establish finitism in the sense of \((b2)\); and the truth of finitism in the sense of \((b1)\) does not necessarily justify finitism in the sense of \((b2)\). This indicates that the claim of Craig’s opponent—i.e., that his argument for the finitude of the past is easily transformable into an equally powerful argument for the finitude of the future—is groundless.\(^\text{19}\)

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