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IS AN OPEN INFINITE FUTURE IMPOSSIBLE?  
A REPLY TO PRUSS

Elijah Hess and Alan Rhoda

Alexander Pruss has recently argued on probabilistic grounds that Christian philosophers should reject Open Futurism—roughly, the thesis that there are no true future contingents—on account of this view’s alleged inability to handle certain statements about infinite futures in a mathematically or religiously adequate manner. We argue that, once the distinction between being true and becoming true is applied to such statements, it is evident that they pose no problem for Open Futurists.

In a recent article, Alexander Pruss has argued on probabilistic grounds that Christian philosophers should reject Open Futurism, i.e., the view that propositions of the form “X will obtain,” where X’s occurrence is not determined by the present state of the world, are not true. To make his case against Christian Open Futurism, Pruss asks us to imagine a possible world in which it is guaranteed that (a) the past is finite, (b) the future is infinite, and (c) every day an indeterministic and fair coin is tossed. Given these background assumptions, Pruss argues that, according to the Law of Large Numbers, a proposition such as “The coin lands heads infinitely many times” will have a probability of nearly 1, a term he defines disjunctively as “either 1 or 1 minus an infinitesimal.” Letting q stand for the above proposition, Pruss goes on to claim that this implication of probability theory poses a problem for the Open Futurist.

The Open Futurist’s view commits her not only to the claim that q isn’t true, but also to the claim that q never becomes true. For there is always a causal

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1 Either because all such propositions are false or because they lack truth value.
2 The Law of Large Numbers is a statistical theorem which says that, as the number of identically distributed, randomly generated variables increases, their sample mean (average) approaches their theoretical mean. For example, when a fair coin is flipped once, the theoretical probability that the outcome will be heads is equal to ½. Therefore, according to the Law of Large Numbers, the proportion of heads in a large number of coin flips should be ½. In particular, the proportion of heads after n flips will “almost surely” converge to ½ as n approaches infinity. Cf. Siegmund, “Probability Theory.”
possibility that there will be only finitely many heads. And yet $q$ has a probability of nearly 1. How can one believe that a proposition with probability nearly 1 is neither true nor becomes true?  

To make the problem even sharper, Pruss considers $q^*$, the proposition that an indeterministic and fair coin is tossed on every day of a time sequence that goes on forever and lands heads on infinitely many of these days. Supposing that some coin is guaranteed to be tossed in this manner, Pruss notes that, like $q$, the probability of $q^*$ will be nearly 1. “But on an Open Future view,” he says, “it is impossible that the proposition $q^*$ ever be true. For, necessarily, on every day of every time sequence, $q^*$ is not true, since if $q^*$ were true, there would be a fact about future contingents, namely that the coin will land heads infinitely often.” Thus, a conflict emerges. The Law of Large Numbers says that $q^*$ is nearly certain. But, given Open Futurism, $q^*$ can never be true. It is implausible to reject the Law of Large Numbers, so Pruss recommends that the Open Futurist’s best bet would be to reject the possibility of an infinite future.

Rejecting the possibility that the future could be infinite, however, is a bet that Pruss informs us a Christian philosopher can’t take. For it is a Christian dogma that there be a future resurrection, one where at least some will partake in an everlasting—and hence infinite—life of union with God. As such, a Christian philosopher should not deny the possibility of an infinite future. Pruss therefore concludes that a Christian philosopher should not believe in an Open Future.

This is an intriguing argument. Given the above coin toss scenario, the Law of Large Numbers says that the following outcome is nearly certain:

*The coin will land heads infinitely often.*

This, essentially, is the proposition Pruss calls $q^*$. So, given Pruss’s coin toss world, plus standard probability theory, we get:

1. $q^*$ has a probability of nearly 1.

But according to Open Futurists, since $q^*$ is a future contingent it can’t be true. Hence, Open Futurists believe that

2. $q^*$ cannot be true.

The fundamental point Pruss wants to make, then, is this: *it is irrational to believe both (1) and (2).* Since he takes (1) to be unassailable, Pruss thinks a Christian who believes in an infinite future ought to give up (2) and, thus, ditch Open Futurism.

Despite its elegance, there are two ambiguities in the above argument that need to be resolved in order to evaluate it. First, according to the coin toss scenario envisioned by Pruss, the future is supposed to be infinite. But what sort of infinity is in view here? After all, the future may be infinite

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in one of two ways. It could either be actually infinite or merely potentially infinite. As it stands, Pruss’s formulation of \( q^* \) strongly implies the former (an indeterministic and fair coin “lands heads on infinitely many . . . days”). With respect to the future, though, Christian Open Futurists such as ourselves hold to the latter conception of infinity. Consequently, Pruss’s initial way of framing the issue is problematic, for it isn’t done in a manner that his interlocutors can immediately accept. Indeed, over and against so-called eternalist theories of time, where all events—whether past, present, or future—are equally real, we maintain that there is a deep ontological asymmetry between the present and the future. Unlike the present, the future does not exist. So while, as Christians, we are committed to the notion that our days in heaven are everlasting and thus will have no end (Luke 1:33), we deny that the future is comprised of an actual infinite series of days, each standing in an earlier than relation to another.

As it turns out, Pruss recognizes the problem. “Talk of how many times the coin will land heads over an infinite,” he admits, “makes it sound like there actually might be an infinite number of future heads tosses.” However, he goes on to say that the scenario can be formulated without any such worries. To accommodate views like ours, then, Pruss notes that, on the assumption that time is linear, the claim that the future is infinite can be put like this: “[T]omorrow there will be a day, and after every day there will be another day.” And, assuming this claim about an infinite future, the claim that the coin will land heads infinitely many times can be put like this: “There will be at least one heads landing, and some time after every heads landing there will be another heads landing.”

More specifically, since the majority of his Open Futurist opponents are presentists, the world described in Pruss’s thought experiment ought to be understood as a place where only the present exists, while the past and future do not. Hereafter, we’ll call this presentist-friendly conception of Pruss’s coin toss world \( W \). In order to avoid begging the question against Open Futurists, therefore, \( q^* \) needs to be formulated in a way that is consistent with \( W \). That is, \( q^* \) needs to be phrased in a way that doesn’t imply an actual infinite or the existence of any future coin toss. In keeping with Pruss’s suggestion above, we submit that a sufficiently neutral formulation of \( q^* \) can be put like this:

\[ q^* N : \text{For any natural number } n, \text{ there will occur some time after the } n\text{th coin toss another toss which lands heads.} \]

In other words, we affirm that the future is ontically open rather than ontically settled or closed. According to Alan Rhoda, the future is ontically open relative to time \( t \) if and only if the world state at \( t \) does not stand in an earlier than relation to a unique and complete series of subsequent world states (Rhoda, “The Fivefold Openness of the Future,” 73).


Though, an Open Futurist could also hold to a growing block view of reality (roughly, the view that both the past and present are real, but the future is not) without affecting the argument being made here.
With this revision in place, (1) and (2) then become

(1*) \( q^*N \) has a probability of nearly 1.
(2*) \( q^*N \) cannot be true.

Now, the question before us is whether it is indeed irrational to believe both (1*) and (2*). But before arriving at an answer there is a second ambiguity in Pruss’s argument that needs to be cleared up. We need to know why Pruss thinks it would be irrational to believe these claims. Perhaps the idea is that there’s an incoherence here. Initially at least, it is tempting to think that what underlies the perception that (1*) and (2*) are in conflict is that, on the one hand, (1*) seems to be intended as a claim about the probability of (1*)’s being true, viz. (1’):

(1’) The chance that \( q^*N \) is true is nearly 1.

On the other hand, though, (2*) seems to entail (2 ’):

(2’) The chance that \( q^*N \) is true is zero.

Obviously, (1’) and (2’) are in conflict. The chance that \( q^*N \) is true cannot be both nearly 1 and 0.\(^{10}\) However, we’re not at all confident that this reading of (1*) and (2*) captures the difficulty Pruss has in mind. For while he undoubtedly thinks there is a tension between (1*) and (2*), it’s not clear that Pruss intends for them to be read as contradictory claims. More modestly, all Pruss’s argument requires is that, from an epistemic standpoint, they appear to clash. The challenge for a Christian Open Futurist, then, is to explain why—contrary to appearances—it would be rational to believe both claims in \( W \). Specifically, such an Open Futurist needs to provide an alternative reading of either (1*) or (2*) that massages the perceived tension and, at the same time, upholds both the possibility of an endless future and the Law of Large Numbers. Our conviction is that this can be done once we observe a distinction inherent to Open Futurist metaphysics: namely, the distinction between the probability of a certain proposition’s being true and the probability of its tenseless content becoming true or coming to pass.

To begin, consider claim (2*). Understood along Open Futurist lines, (2*) should be read like this:

(2^) The chance of \( q^*N \)’s being true is zero.

Why should an Open Futurist say that? Because the chance that a proposition is true can only be either zero or one. It’s one if the proposition is, in fact, true. It’s zero if the proposition is, in fact, not true.\(^{11}\) And, of course,

\(^{10}\) Where “chance” = a single-case, objective probability.

\(^{11}\) Some may wonder why we distinguish between (2’) and (2^) since both seem to be identical interpretations of (2*). The reason we focus on the (2^) reading of (2*) is simply to make explicit the concept of something’s being the case. This is done in order to set up a contrast with another concept that we introduce below, that is, the concept of something’s becoming or coming to be the case.
the Open Futurist maintains that there can be no true determinate future contingent propositions.\textsuperscript{12} In particular, Christian Open Futurists such as ourselves who accept the principle of bivalence maintain that $q^*N$ is false in Pruss’s coin toss world. If it were true, we believe God would know this. But given the metaphysics of Open Futurism, God does not know $q^*N$ at any time.

The idea that God would not know $q^*N$ is predicated on the plausible assumption that God only believes on sufficient evidence. According to Christian Open Futurists, God is temporal and hence exists moment-by-moment in $W$. Thus while it is no problem for God to know that for any natural number $n$, there will be more than $n$ coin tosses (according to Pruss’s thought experiment, this is guaranteed to happen in $W$), such a deity would presumably not believe that for any $n$, there will be more than $n$ heads landings. That is, God would not believe $q^*N$. The reason we say this is because, in addition to their belief in a non-actual future, Open Futurists subscribe to the metaphysical doctrine that contingent truth depends upon being. Hence, we maintain that there is not enough “being” at any given moment in $W$ for there to be a determinate fact of the matter concerning the outcome of any coin toss that has yet to take place. So, although the prospect that there will be at least one heads landing, and after every heads landing there will be another heads landing is extremely likely in $W$, there is nevertheless a vanishingly small chance that there will only ever be tails landings after any given toss. Ontologically, then, for any present time $t$, the possibility that there will only be tails landings after $t$ cannot be definitively ruled out. As a result, an infallible, omniscient being that exists temporally in $W$ wouldn’t believe $q^*N$.\textsuperscript{13} By his very nature, God only believes that which he is certain of. Given the metaphysics of Open Futurism, however, God lacks sufficient evidence to be certain of $q^*N$. It therefore follows that $q^*N$ isn’t true at any time in $W$. This is why,

\\textsuperscript{12}The word \textit{determinate} is important here. For, in the context of the present discussion, the term “future contingents” is really shorthand for what we might call “representationally determinate propositions about the future,” i.e., propositions that represent the future as determinate in some respect. This is normally expressed in English by saying that some event unqualifiedly “will” or, alternatively, “will not” happen. This contrasts with propositions saying that some event “might and might not” or “probably will” happen. In the latter case, the future isn’t represented as being determinate with respect to that event.

\textsuperscript{13}Given that $q^*N$ has a probability of nearly 1, it might seem fantastic to think that God would refrain from believing such a proposition on account of there being an infinitesimal chance that the event described therein \textit{not} occur. After all, since an infinitesimal number is a number that is smaller than any positive real number but greater than zero, God, it could be argued, would hardly register such a miniscule probability. However, we think this is a mistake. As one who is perfect, it is plausible that God’s cognitive powers are sufficiently fine-grained to track infinitesimals. Moreover, being infallible, divine certainty leaves no room for error. So long as the theory is consistent, then, it seems reasonable to suppose that God knows and responds to infinitesimal probability values. For a recent defense of the coherence and utility of such probability values see Benci, Horsten, and Wenmackers, “Infinitesimal Probabilities.” Cf. Robinson, \textit{Non-standard Analysis}. 


according to \((2^*)\), the chance of \(q^*N\)’s being true at any time in \(W\) is zero—which is just to say that \(q^*N\) cannot be true there, precisely as \((2^*)\) claims.

So, belief in \((2^*)\) seems perfectly sensible on the Christian Open Futurist’s metaphysic. But wouldn’t it be irrational for such a philosopher to believe both \((2^*)\) and \((1^*)\), as Pruss suggests? No. For when properly understood, \((1^*)\) does not conflict with \((2^*)\). To demonstrate this, consider a simpler probabilistic argument Pruss once gave against Open Futurism.\(^{14}\) We’ll cite Pruss’s restatement of this argument, as contained in his more recent article. He writes,

> Suppose that I am determined by the present conditions and laws of nature to flip an indeterministic fair coin in exactly five minutes. According to Open Future views, it is neither true that the coin will land heads nor that it will fail to land heads (either both statements are false or neither statement has a truth value). Yet by definition of fairness, the probability that the coin will lands heads is 1/2. So the Open Futurist has to believe both that it is not true that the coin will land heads and that the probability that it will land heads is 1/2. Yet surely if one believes that it is not true that the coin will land heads, one assigns a probability less than 1/2 to the proposition. We can make the problem sharper by supposing the coin to be unfair and to have a probability of 9/10 of landing heads. Then the Open Futurist has to believe both that it is not true that the coin will land heads and that it has a probability 9/10 of doing so.\(^{15}\)

Yet, Pruss now concedes that the Open Futurist has a way out of this problem. In light of a reply provided by Alan Rhoda,\(^{16}\) Pruss now acknowledges that

> The Open Futurist can, for instance, say that there is a tenseless proposition, \(u\), that the coin lands heads at \(t_5\), where \(t_5\) is five minutes from now. The sentence “The coin will land heads in five minutes” can be said to have \(u\) as its “tenseless content” . . . Then the probability that the coin will land heads in five minutes is \(r\) because \(u\) has a chance of degree \(r\) to become true. In other words, claims about the probabilities of future contingents are claims about the chances-to-become-true of tenseless propositions that are at present [not true].\(^{17}\)

We contend that this same line of response can be adopted to show why \((1^*)\) does not, in fact, conflict with \((2^*)\). For consider that, no matter what number we plug into the statement schema we’ve been calling \(q^*N\), the event described there will have a near certain chance of coming to pass. For example, suppose the coin has just been flipped for the 56th time in \(W\). As Pruss will concede, the Open Futurist can maintain that there is a tenseless proposition, \(u\), that the coin lands heads at some time \(t\), where \(t > 56\). The sentence “There will occur some time after the 56th toss another toss which lands heads” can be said to have \(u\) as its tenseless content. Then the probability that the coin will land heads at \(t\) is \(r\) because \(u\) has a chance of degree \(r\) to become true. In other words, from an Open Futurist’s perspective,

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\(^{14}\)See Pruss, “Probability and the Open Future View.”

\(^{15}\)Pruss, “An Open Infinite Future is Impossible,” 461.

\(^{16}\)Rhoda, “Probability, Truth, and The Openness of the Future.”

\(^{17}\)Pruss, “An Open Infinite Future is Impossible,” 461–462.
claims about the probabilities of future contingent events are claims about the chances-to-become-true of tenseless propositions describing these events that are at present not true. As such, (1*) should be read like this:

\((1^\wedge)\) The chance of \(q^*N\)'s descriptive content coming to pass is nearly 1.

By “descriptive content” we mean the event description that is specified by substituting a natural number for \(n\) in the schema “There will occur some time after the \(n\)th toss another toss which lands heads.” As noted above, this schema can be said to have the following tenseless content: The coin lands heads at some time \(t\), where \(t > n\). Once \(n\) is specified, standard probability theory ensures that the event described by \(q^*N\) has a near certain chance of taking place given a potentially infinite or endless series of tosses. Accordingly, \(q^*N\) is an excellent prediction to make in \(W\), and we would be well advised to bet in its favor for each natural number \(n\). However, no matter how many of these bets are successful the truth of \(q^*N\) is not entailed.\(^{18}\) For, again, there will always be an astronomically small (though nonzero) chance that there will only ever be tails landings after any future toss.

The upshot is that there is no real conflict between (1*), understood as (1^\wedge), and (2*), understood as (2^\wedge). Thus, we conclude that Pruss’s argument should not dissuade a Christian philosopher from believing in an Open Future.\(^{19}\)

References


\(^{18}\)Thanks to William Hasker for this point.

\(^{19}\)We’d like to express our appreciation to both the editor and two referees for their helpful feedback on earlier versions of this paper.