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A PARADOX OF OMNISCIENCE AND SOME ATTEMPTS AT A SOLUTION¹

Alfred J. Stenner

A paradox is constructed employing four languages L1-L4, such that L1 is a metalanguage for L3, L3 for L2, and L2 for L1; L4 functions as the semantic meta-metalanguage for each of L1-L3. The paradox purports to show that no omniscient being can exist, given that there is a set of true sentences (each true within its respective language) from L1, L2, and L3 that no omniscient being can believe.

The remainder of the paper consists in an examination of some attempts at challenging the paradox on syntactic, semantic and pragmatic grounds. Just which of these attempts are the most promising for the religious person is a question which is left open.

I

The late Henry Leonard liked to titillate his graduate students with the following question: on the assumption that God is omniscient, is the expression 'God believes that _____' a truth-functional operator? Since a truth-functional operator is one that operates on one or more sentences to produce another sentence whose truth is solely dependent on the truth-value(s) of the former, the question, at first blush, appears to have a relatively straightforward answer. If x is omniscient, then x believes that p if and only if p , for all values of x and all substituends of ' p .' Formally,

$$S1 \quad (x) (Ox \rightarrow (Bxp \leftrightarrow p))$$

which says no more than that an omniscient being believes all and only what is the case. Assuming 'Og' ('God is omniscient'), it follows from S1 that

$$S2 \quad Bgp \leftrightarrow p$$

for all substituends of ' p .' If we assume that a given substituend of ' p ' is false, we write ' $\sim p$.' This, together with S2 yields, ' $\sim Bgp$ ' (i.e., 'It is not the case that God believes that p '). Moreover, if we substitute ' $\sim p$ ' for ' p ' in S2 the result is

$$S3 \quad Bg\sim p \leftrightarrow \sim p$$

S2 and S3 together yield

$$S4 \quad Bg\sim p \leftrightarrow \sim Bgp$$

So at first blush, it appears that 'God believes that' is a truth-functional operator for regimented English sentences, provided that God is omniscient. If p is true, then and only then does God believe that p ; for an omniscient being does not believe what is false. On the other hand if p is false, then it is not true that God believes that p and God believes that p is false. So God believes that p if and only if p . It would appear then that the crux of the matter is whether or not God is omniscient. It is the purpose of this essay to examine a paradox which purports to show that God is not omniscient; or, rather, that there is no omniscient being. Consider the sentence

(A) $\sim$ (God believes that (A) is true)

The argument purports to show that (A) is a true sentence.² To make the following deduction clear let us note that '(A)' is a name for the sentence 'God does not believe that (A) is true.' [I use 'A' as an abbreviation of (A).] The argument proceeds by way of a reductio.

1	$\sim A$	assumption
2	$\sim\sim$ God believes that (A) is true	by replacement of 'A' in 1
3	God believes that (A) is true	2, double negation
4	God believes that (A) is true $\leftrightarrow$ (A) is true	replacing 'p' in S2 with '(A) is true'
5	(A) is true	3,4
6	A if and only if (A) is true	principle of semantics
7	A	5,6
8	$\sim A \rightarrow A$	1-7 by conditional proof
9	A	from 8
10	(A) is true	From 9 and same principle as employed in 6

Hence, although (A) is true, God does not believe it is true. Hence God is not omniscient. He fails to believe at least one true claim.

Since the sort of explicit self-reference employed in (A) may be thought to be unacceptable in the formulation of the paradox let us rewrite (A) as

(A') God does not believe that the sentence appearing on lines 89 and 90 of Stenner's word processor in the file headed by 'Noter13' is true.

Anyone who takes the trouble to go to that particular file will discover that (A') is the sentence which appears on lines 89 and 90 of Stenner's word processor in the file headed by 'Noter13.' So it follows that God does not believe that (A') is true. It follows that the paradoxical consequence follows just as surely from this revised version as from the first.

Those familiar with Tarski's work may immediately object that the paradox

is dependent on a faulty assumption, namely, that it is possible for a language to contain its own semantical principles. The truth predicate for a language (if that language is to be free of semantical paradoxes) must belong, not to that language, but to a semantical metalanguage in which we can describe the semantical properties of that language (that is, the object language).

We might, therefore, attempt to reconstruct the paradox by simply removing the phrase 'is true' so that we now get

(A'') God does not believe the sentence appearing on lines 108 and 109 of Stenner's word processor in the file labeled 'Noter13'.

Again the paradoxical consequence follows.

But an objection to this formulation might be that I am cheating; for the sentence implicitly says of itself that God cannot believe that it is true. While I am not quite sure of what "implicit saying" comes to I will not press this point. Rather I want to show that another argument can be constructed employing three languages, L1, L2, and L3 related as follows: L2 is the semantic metalanguage for L1, L3 is the semantic metalanguage for L2, and L1 is the semantic metalanguage for L3. Consequently, the truth predicate for each of these languages appears in its respective metalanguage. (I shall not at this point go through the mechanics of formulating formation rules for the respective languages but will assume that this can be done with little difficulty in the traditional way.) Now consider the following set of statements:

1. No omniscient being believes that sentence 3 is true-in-L3.
2. Sentence 1 is true-in-L1.
3. Sentence 2 is true-in-L2.

Not only does this set of statements not violate any clearly defensible semantical principles, but each of the sentences is true in its respective language! We argue as follows: Suppose 1 is false-in-L1. Then some omniscient being believes that sentence 3 is true-in-L3. But then such a being must believe that sentence 2 is true-in-L2 since sentence 3 says it is true-in-L2. Hence the omniscient being in question must believe that sentence 1 is true-in-L1 since sentence 2 says it is true-in-L1. Sentence 1 must then be true since it is believed by an omniscient being to be true. But then our initial assumption that some omniscient being believes that 3 is true-in-L3 is false. Hence 1 is true-in-L1. There are thus true sentences which no omniscient being believes to be true. The argument is perfectly general with respect to omniscient beings and in no way depends on the assumption that God is omniscient. Consequently the conclusion is that there is no omniscient being. Strangely enough, each of us non-omniscient beings can believe that 1 through 3 are one and all true in their respective languages.

II

In the remainder of this paper I examine various ways the paradox can be challenged and determine the effectiveness of such challenges. In this section I want to consider some objections which trade upon certain logico-semantic features of paradoxes. First, one might attempt to argue that the system of languages, L1-L3, constitutes a circular set and that it can easily be shown that such circularity can result in semantic paradox just as easily as can a simplistic version of the liar's paradox. For example,

- A. Sentence C. is true-in-L3.
- B. Sentence A. is true-in-L1.
- C. Sentence B. is false-in-L2.

To show that this set of sentences and hence the set of languages of which they are a part is inconsistent we simply show in a metalanguage, L4, containing L1-L3 as sublanguages, that sentence B is both true-in-L2 and false-in-L2. We argue as follows: If B is true-in-L2 then A is true-in-L1. And if A is true-in-L1 it will follow that C is also true-in-L3 and B is false-in-L2. So if B is true-in-L2 it is false-in-L2. But then it is false-in-L2. But if it is false-in-L2 then so is A false-in-L1 and C false-in-L3. But if C is false-in-L3, then B is true-in-L2. So if B is false-in-L2, then it is true-in-L2; and hence it is true-in-L2. Sentence B is then both true-in-L2 and false-in-L2. So insofar as the languages L1, L2, and L3 function as semantic metalanguages in this circular fashion we have shown that they are inconsistent. It follows that any set of languages constructed in such a circular fashion can be shown to be inconsistent in precisely the way in which L1, L2, and L3 constitute an inconsistent set. It is not surprising therefore that sentences 1-3 should result in the conclusion that there is no omniscient being since the languages in which they are formulated will turn out to be inconsistent.

I think that this argument is based on a flawed analogy between the case of an omniscient being and the case just mentioned. The two cases are actually quite different, the first one involving a veridical paradox, the second a falsidical paradox.³ The first consists of demonstrably true sentences, the second gives rise to a contradiction and consequently not all of the sentences A, B, and C are true. The conclusion to the first of the paradoxes is surprising but it remains to be seen whether this paradox of omniscience contains a logical flaw or employs an unsatisfactory principle of inference. What I am arguing in particular is that the paradox generated by sentences 1-3 is entitled to be classified with the veridical paradoxes such as Godel's incompleteness results rather than with the antinomies such as the liar's paradox. Whereas A, B, and C are demonstrably incapable of all coming out true together, it is demonstrable that 1, 2, and 3 are

not merely jointly consistent but that each is true! Moreover, if an omniscient being is one who knows of each true sentence of a language that it is true in that language, then there are no omniscient beings. For no omniscient being can know that each of sentences 1, 2, and 3 is true in its respective language, assuming that warranted belief is a necessary condition for knowledge.

For the sake of the argument, however, let us suppose that it is possible to generate an inconsistency from the sentences of languages L1, L2, and L3. The question then is this: how does this fact have any bearing on the argument concerning omniscience? Ordinary English can be shown to be inconsistent by a similar set of arguments. But this does not prevent us from saying true things in English, even true things about the semantic properties of English sentences. What Tarski showed is not that we cannot say anything about the semantics of English in English, but merely that if we try to formulate a *complete semantic theory for English in English*, then paradox can be generated in that language. Similarly if we try to construct L1 in such a way as to provide a complete semantical metalanguage for L3, and try to do the same for L1 and L2, we shall be able to generate the kind of paradox noted above with respect to A-C. But our argument for the non-existence of an omniscient being need not make such presuppositions at all. For example, we may so construct L1 that it contains only sentences asserting what individuals believe about the sentences of language L3. I doubt that any contradiction, such as the version of the liar's paradox just remarked, could be generated in such a triad of languages although I have no knock-down argument to prove it. The important point, however, is that I have just shown that the three sentences which establish the nonexistence of an omniscient being are one and all true in their respective languages. So even though a semantic paradox could be generated in such a triad of languages this possibility in itself cannot be grounds for holding that every semantic claim made within these languages is false or paradoxical.

Another method of criticizing the paradox is to attack the argument for the conclusion by noting that I have failed to specify the formation rules for L1 through L3 and concluding that it is not at all clear whether the three languages constitute an inconsistent triad or not. Moreover, until I provide a set of formation rules for each of the elements of the triad, I am in no position to argue that semantic paradox is not forthcoming from the triad. This objection can be rebutted by noting that L1 through L3 consists of just a subset of sentences from regimented English. Insofar as we understand English we can understand these English sentences and "grasp" what they say. My argument (which presumably takes place in L4, another subset of sentences from regimented English) supposes that these subsets can be specified in a consistent way. As long as the sentences of L1 are limited to statements of belief, I see no way in which the sort of contradiction generated with respect to sentences A, B, and C can be generated in the

case of languages L1-L3, provided that L1 be restricted to statements about what rational agents can believe about statements formulated in sentences of L3. Of course this may be just blindness on my part. But even if such contradictions can be generated, this presumed state of affairs has no obvious bearing on the question whether the *paradox* in question involves a contradiction and whether such contradiction is somehow dependent on the question of the formation rules for L1 through L3.

III

A second objection might go as follows: given the referential opacity of belief contexts, it is not immediately obvious whether the appropriate semantic connections can be developed formally in languages like L1 through L3.⁴ Although a complete analysis of belief contexts would surely be a welcome philosophical breakthrough, nothing of this order seems to be required in the present circumstances. What is needed is justification for the schema, S1, which “unpacks” the notion of an omniscient being to the extent of specifying a necessary condition for the term ‘omniscient being’ to be applicable at all. It simply requires that the result of replacing ‘p’ in S1 by some true or false regimented sentence of English will always result in a true generalization. Since an omniscient being is generally held to believe only what is true, the aforementioned requirement seems hardly to be objectionable. One might choose to specify knowledge rather than belief as a necessary condition for omniscience on the ground that an omniscient being is to be characterized as *knowing* what is true rather than merely believing what is true. A paradox similar to the one generated by sentences 1 through 3 can be created by replacing ‘believes that’ by ‘knows that.’ But if belief constitutes a necessary condition for knowledge then this particular route seems unnecessary. Suffice it to say that the generation of the paradox clearly depends on the adequacy of S1 and no more appears to be required by way of analysis of belief contexts, for current purposes, than its justification.

Before turning to other issues, I wish to make a few additional comments concerning the semantic principles employed in the languages L1-L3. One of Tarski’s strictures, i.e., not allowing a language to contain all of its own semantic principles, is clearly not something that one *discovers* by an empirical investigation.⁵ Rather we make a *decision* to use a certain linguistic framework for whatever purposes we have in mind. In Russell’s case the decision was made in order to avoid certain paradoxes. We currently adopt Tarski’s methods for similar reasons—the avoidance of paradox and, more generally, avoidance of inconsistency deriving from whatever semantic source. It therefore would fall to those who wish to hold that there is in fact one omniscient being to show that L1-L3 are languages in which it is not possible to carry on cognitive inquiry or

that true statements cannot be made in these languages and that hence there is a critical flaw in the argument presented in this paper. Nor will it do to claim that sentences 1 through 3 are cognitively meaningless, i.e., neither true nor false, for I have just shown that there appears to be a sound argument to the conclusion that all three of the sentences are true.⁶

I have assumed, throughout this discussion that what 'belief' means as applied to omniscient beings is the same as when applied to ordinary mortals. Although extended discussion of metaphorical and analogical language as applied to deity is not possible here⁷, there are some general comments concerning the analysis of belief contexts as applied to deity which should be made with respect to this assumption. The expression 'believe' does not wear its logical syntax on its sleeve; consequently, there are several options available to us. We can treat belief as a relation, or treat 'believe' as part of a logical operator or as a part of a predicate. In discussing the problem of the logical syntax of belief sentences I shall continue in this paper to adopt the view that 'belief' and its cognates as applied to deity are best understood in the literal sense of the term. Following suggestions made by Israel Scheffler⁸ concerning a way of analyzing belief sentences, I begin by examining the following:

God believes gold is malleable if and only if God believes-true the sentence 'Gold is malleable'.

Here 'belief' is not treated as a relational term but rather as a part of a relational term, 'believes-true,' a term which holds between persons and sentences of a language. Scheffler thus is able to bypass some of the thorny psychological issues involving belief and believing. One objection that has been raised to this sort of analysis, at least as regards ordinary mortals, is that it requires a conditional rather than a biconditional "definition," namely, "If P believes-true the sentence, 'Gold is malleable,' then P believes gold is malleable." The converse fails for those who are unable to read, speak, or understand English. Scheffler's analysis then provides only a sufficient and not a necessary condition for belief sentences. But this criticism is irrelevant to our inquiry since an omniscient being believes-true all and only the true sentences of every language.

To avoid needless complications, I consider only eternal sentences of regimented English of the Quinean sort as permissible substituends of 'p.' While biconditionals of the following sort

God believes it is raining if and only if God believes-true the sentence, 'Il pleut'

are apparently harmless, they may lead to needless complications for the discussion and will consequently be ignored.

Another criticism that allegedly threatens Scheffler's analysis is ambiguity. If

an ambiguous sentence occurs as the second relatum of a belief sentence as thus construed, then the sentence's truth-value may depend crucially on whether the sentence is construed as true or construed as false. This difficulty may be remedied by considering only belief sentences involving the deity in which an eternal sentence constitutes the second relatum.⁹ Although Scheffler's account may prove to be unsatisfactory as a general analysis of belief sentences, for the sorts of problems addressed in this paper it appears to be perfectly adequate, since none of the standard criticisms have any bearing on the problems at hand.

I think it is clear, therefore, that the claim that the paradox is due somehow to a faulty (because unexplicated) notion of *belief* is seen to be without basis.

IV

In this section I examine an attempt to save the doctrine of God's omniscience from the paradoxes by distinguishing between knowing that *p* and knowing that someone knows that *p*.

Something like omniscience can perhaps be rescued from the paradox of omniscience by considering that there does not seem to be anything incompatible with God's knowing that each of us (who does in fact know that 1-3 is true) knows that 1-3 is true, unless we assume that the following principle holds universally:

If A knows that B knows that *p*, then A knows that *p*

But this principle can be shown to be untenable if assumed to be held without qualification.¹⁰ Consider the following sentence

D. A. J. Stenner III cannot know that this sentence is true.

While I am incapable of knowing that D is true, I am assuming that you know it is true even though you may never have met the author of this essay. So it appears that I know that you the reader knows that D. is true. But it is clear that I do not know that D. is true even though "I know that D" follows from the transitivity principle and the claim that I know that you know that D. Yet I seem to know something about D. What is it that I know? What is interesting is that I seem to be able to know something which I clearly cannot know. There is a puzzle here and it is a puzzle which also afflicts the paradox of omniscience.

What these remarks seem to suggest is that there are two distinct senses of 'know,' one in which I cannot know that D. is true and one in which I can know it. The one way we may refer to as *direct* knowledge and the other as *indirect* knowledge. Similarly, although an omniscient being cannot directly know that the troublesome sentence is true, an omniscient being can indirectly know it via the transitivity of the knowledge relation (if indeed the knowledge relation is

transitive in the sense intended). What this suggestion amounts to is this: if A directly knows that B directly knows that p , then A indirectly knows that p . We may now attempt to characterize omniscience as follows:

x is omniscient if and only if for every true statement, S , x directly knows that S is true or x indirectly knows that S is true.

We can now characterize knowing as direct or indirect. So a shortened version of the definition of omniscience will be

x is omniscient if and only if for all true statements, S , x knows that S is true

where 'know' is understood as characterized above. Before we examine the question whether omniscience is logically possible and whether the doctrine of God's omniscience can be rescued from the paradoxes, there are some other complicating factors to be taken into account. First of all, there are infinitely many paradoxes of the sort we have been considering. To establish this we need note only that the circle of languages involved can be expanded to n languages, for any positive integer, n . Of course not all of these have been actually inscribed and they exist only in the sense that, analogously, there are infinitely many sentences in English and infinitely many truth-functions in the truth-functional logic. Now the problem is this: the notion of indirect knowing depends on there being a person (rational agent), P , other than God, who directly knows the statement in question is true. Assuming that there is some finite upper bound to the number of languages a finite rational being can handle and comprehend, there will always be a paradox involving a number of languages greater than any finite rational being could handle and comprehend. It would seem to follow that (past, present, and future) there will be infinitely many paradoxical statements of the sort under consideration that God cannot know even in the indirect sense since there are no rational agents who directly know that the paradoxical statement is true.¹¹ There are two immediately obvious ways out of this difficulty, each carrying a price tag. One way requires the use of counterfactual conditionals. God indirectly knows S is true just in case if there were a rational agent P who knew that S is true, God would directly know that P knew that S is true. I perhaps need not comment on the difficulties attending counterfactual specification of necessary and sufficient conditions.

A second way (to which the first way is reducible on accounts such as those of David Lewis)¹² requires the existence of possible beings who, in their own possible worlds, directly know that S is true. On this account God indirectly knows that S is true provided there is a possible rational agent, P , who directly knows that S is true. Thus we need to distinguish between

- (i) In w , P knows that Socrates failed to drink the hemlock in w

and

(ii) In w , P knows that Socrates drank the hemlock in the actual world.

where 'w' denotes a possible but non-actual world. An agent that knows (i) and (ii) presumably does not have contradictory beliefs. Both (i) and (ii) are true. As far as the paradoxes are concerned, however, only (ii) is relevant. The way we want (i) and (ii) to cash out is that if God knows that (i) is true, God knows indirectly that it is possible that Socrates failed to drink the hemlock.¹³ And if God knows (ii) is true, then God indirectly knows that Socrates drank the hemlock.

Have the paradoxes been circumvented? Unfortunately not. It can easily be proved that this modal retreat is inadequate as a way out. To see that this is the case let us first define 'know' as follows:

Def. x knows that p = df. x indirectly knows that p or
 x directly knows that p

The proof at the end of section I of this paper may be reconstructed by replacing 'believes' (or a grammatical variant thereof) wherever it occurs in that proof with 'knows' (or by appropriate grammatical variants) beginning with its occurrence in sentence 1. The proof goes through as easily as does the original as the reader may attest by carrying out the reconstruction.

The consequence then is that there are no omniscient beings since

1'. No omniscient being knows that sentence 3 is true-in-L3.

So the distinction between direct and indirect knowledge has not enabled us to avoid the paradoxical conclusion.

V.

Since our logico-semantic excursus has not provided a satisfactory solution to the paradox of omniscience let us turn our attention in a different direction. There is a feature of paradoxes of omniscience which is shared with some of the paradoxes of omnipotence. It is this: in each case the paradox is created by devising a game which the other player cannot win except by cheating or by trading upon some disability (e.g., exhaustion) of the other player. Suppose we ask, "Can God win a game of tick-tack-toe against a knowledgeable player?" Before we try to answer this question let us look at some of the complicating factors. First of all, since God neither slumbers nor sleeps, He has an advantage over ordinary players who, because of their humanity, are in need of sleep, food, drink, etc. If it were possible for an omnipotent being to play tick-tack-toe with a finite rational being, the obvious answer to the question is "Yes." All the

omnipotent being would need to do in order to win is to wait until the opponent dropped from exhaustion, was awakened to make the next move and made a stupid blunder of the sort that usually accompanies such disability. But this surely says nothing for the omnipotence (or lack of it) for one of the players. Let us then rather construct a scenario of a slightly different sort. The question then is, "Can an omnipotent being win a game of tick-tack-toe against a machine (such as a computer) programmed to play the best strategy and kept in optimal working condition?" The answer to this question is clearly in the negative. The phrase 'kept in optimal working condition' in the phrasing of the question is important; for it prevents the pulling of the plug, for example, as a device for winning. What we want is an ethical strategy for winning of the sort appropriate to deity. So long as the opponent is omnibenevolent, or at least meets the standards for omnibenevolence as is necessary to play the game in question, there can be no winner, since tick-tack-toe is a game for which there can be no winner if both players employ the optimum strategy.

There is a clear analogy here to those paradoxes of omnipotence (e.g., the paradox of the stone) in which the conditions of "contest" are such that there cannot be a winner. In the case of the paradox of the stone the difficulty arises from logical presuppositions in the formulation of the problem. If God's winning in such a case is to create a stone so large that he cannot lift it, then He cannot win. That is, if He creates such a stone, he demonstrates that he is not omnipotent since he has created something over which he lacks control.¹⁴ In the paradox of omniscience there is no faulty presupposition assumed in the formulation of the issues, but it is logically impossible for there to be a winner. Whether this shows that the paradox of omniscience, like the paradox of the stone, has no relevant bearing on the question of divine omniscience (or omnipotence) is something the reader will have to decide.

Those who wish to appeal to the doctrine of analyticity might employ the following formulation of the solution: If the conditions of contest entail there being no winner, then the game in question has no bearing whatsoever on the question of the omniscience of the deity. The claim that God can win at a game of tick-tack-toe under the conditions just described is a counteranalytic proposition. Counteranalyticity, on this view, is just a broader category than that of logical impossibility. And the principle that says that omnipotence remains unaffected by logical impossibility holds likewise for counteranalyticity.¹⁵ Whether this sort of move provides a satisfactory solution to the paradoxes of omniscience of the sort we have just examined is a question I am not at this time prepared to answer. I have not, of course, attempted to answer the question whether God is omnipotent or omniscient. This issue, it seems to me, at least insofar as the analytic-synthetic distinction makes any sense at all, is one that is purely synthetic and demands a synthetic solution.

VI

Two further sorts of moves might be tried in an attempt to avoid the paradoxical conclusion of the argument under discussion in this paper. The first is to deny that sentences have truth-values and argue that it is only propositions or statements which have truth-values. I believe it is perfectly easy to construct the paradoxes on these assumptions but I leave that task to those for whom the enterprise suggests a better solution than the ones I have considered. A second and more important way out involves rejection of the principle of bi-valence. An analysis of the sort I have in mind would result from employing a three-valued logic and semantics. Sentences like 1-3 could then take values other than truth and falsity. The arguments developed in meta-metalanguage, L4, are based on the assumption that the semantics in question are two-valued. Can an argument to the same conclusion be constructed by replacing 'false-in-L1' by 'false-in-L1 or indeterminate-in-L1' where 'indeterminate in L1' simply denotes the third value with similar replacements being made for L2 and L3? While I have no conclusive argument one way or the other, some considerations now lead me to believe that the paradoxes cannot be generated when the semantics of L1-L3 are three-valued. I want to briefly discuss some of the relevant considerations. First of all each of the paradoxes has been generated by the method of indirect proof. This method is not available in a three-valued logic nor is the schema, ' $p \vee \sim p$ ' obtainable as a theorem in those three-valued systems with which I am familiar. I have been unable to discover a proof of the paradoxes which does not depend in some crucial way on the method of indirect proof.

To see why it is unlikely that such a proof can be found, let me take the most direct tack available. I shall employ a two-valued logic in L4 and assume that L1-L3 are languages with three-valued semantics. The "truth-table" for three-valued "negation" is as follows:

p	$\neg p$
T	F
F	T
I	I

Here the third value is construed as indeterminacy. The dash (rather than the tilde) is employed to distinguish this three-valued function from ordinary negation expressed by ' $\sim p$.' The other functions are irrelevant to the argument and won't be defined here. Intuitions which justify this definition of three-valued "negation" are summed up as follows: if a statement is indeterminate then so is its negation,

and *vice versa*. The relations of truth and falsity with respect to determinate sentences remains the same as in two-valued semantics. In order to apply our two-valued logic to the problem at hand we need to employ the tilde instead of the dash, i.e., we need two-valued negation rather than its three-valued counterpart. Unfortunately ‘ $\sim$ ’ is not definable in terms of ‘ $-$.’ But we can characterize ‘ $\sim$ ’ in terms of the three-valued system as follows:

‘ $\sim S$ ’ is true just in case S is either false or indeterminate.¹⁶

We now proceed to construct the argument for the paradoxes as before. The conclusion in the one case is

$\sim(\exists x) (Ox \ \& \ BxS3 \text{ is true-in-L3})$

and in the other case

$\sim(\exists x) (Ox \ \& \ KxS3 \text{ is true-in-L3})$

But the force of the paradox has been clearly diminished; for we can no longer conclude that the claims that $(\exists x) (Ox \ \& \ BxS3 \text{ is true-in-L3})$ and that $(\exists x) (Ox \ \& \ KxS3 \text{ is true-in-L3})$ are each false but only that they are either false or indeterminate.

A slightly different account would read ‘ I ’ as meaning *indeterminable* rather than as *indeterminate*, the former focusing on the limitations of our methods of inquiry, the latter having a more “realistic” flavor. The results, however, are the same: the sharpness of the paradoxical conclusion has been blunted.

What I have shown in this paper is that there are various methods for attempting to cope with paradoxes, more specifically with the paradoxes of omniscience. Some of these, such as those which issue in attacks on the formal or semantic properties of the premises of the argument, are quite wide of the mark and seem to me to be satisfying neither to the philosopher nor to the person of faith. Whether the approach requiring a rejection of the principle of bi-valence is religiously satisfying is a question I shall leave to others.

The question with which I began was this: Is the expression ‘God believes that _____’ a truth-functional operator? As a device for titillation it perhaps succeeds. But in spite of initial reactions to the contrary, and for reasons adumbrated in this paper, theologically inclined logicians are best advised to look elsewhere for a logical basis for their truth-functional theories.

NOTES

1. A slightly different version of this paper was delivered to a philosophy colloquium at Washington University, March 27, 1986. I am grateful to Richard A. Watson for helpful suggestions and for turning some questionable syntactical constructions into English sentences. I also would like to thank the editor of this journal and a referee of this paper for a number of helpful criticisms and comments.

2. We could have taken as our sentence-frame 'God believes that p' with the understanding that the relative 'that' occur as a nominalizing operator on 'p.' In this case 'p' would be treated as a variable ranging over propositions rather than as a schematic letter. The relative 'that' would then be taken as part of the nominalizing expression, i.e., as part of the expression naming a proposition rather than as part of the predicate forming operator 'believes that.' Those who take propositions to be objects of belief may construe 'p' as a quantifiable variable rather than as a schematic letter as I take it to be. There appears, however, to be no particular point in belaboring the distinction between a quantifiable variable and a schematic letter at this point. Whatever interpretation of 'p' we adopt, the argument for the paradox goes through as usual.

3. The terms 'veridical' and 'falsidical' as applied to paradoxes appear to have been introduced by Quine. See the title essay in his *Ways of Paradox* (New York: Random House, 1964).

4. I am unable to provide such an analysis and perhaps it is not beside the point to say a brief word about the difficulties involved. There appear to be two sorts of theories which attempt to explicate belief in a formal way. The first treats beliefs as propositions or meanings, posited to explain the nature of belief and relations among beliefs. The second sort is also of two sub-species: those which keep 'believes' as a relational term and those which treat it as either part of a predicate applied to persons or perhaps part of an operator. Theories which posit propositions and meanings are the results of attempts to avoid the difficulties attendant upon the inscriptional approach of the sort advocated by Israel Scheffler and others. But it is not clear that adopting an analysis of belief contexts which posits intentional entities avoids the possibility that the unwanted conclusion may not arise for other reasons. The inscriptional approach, while avoiding certain problems posed the positing of meanings or propositions, fails to provide a completely general account which allows for differences in languages. For example, it cannot handle even the simplest case of utterances of statements of belief such as 'John Claude believes it is snowing' on the occasion of the French-speaking, non-English-speaking Jean Claude tramping through the streets of Montreal in the midst of a blizzard. In other words, the inscriptional approach is language relative. These are one and all difficult issues and I have no answer to any of them. I am not attempting to ignore the issues raised by the attempts to construct a formal semantic theory which at the same time can handle belief sentences. What I am suggesting is that our inability to construct such a theory is irrelevant to the question of whether we can tell that sentences 1, 2, and 3 are one and all true. Notice that we can construct a similar paradox by rewriting sentence 1 as 1a, namely 'Georg Cantor cannot believe-true that sentence 3 is true in L3' although everyone of the rest of us can clearly tell that the now revised set 1a-3 contains only true sentences. So the paradox is clearly not dependent on the notion of omniscience, except to the extent that omniscient beings are supposed to believe all and only true sentences. A few more comments on the nature of such theories follows shortly in the body of the text.

5. Russell apparently held the view that one could in fact discover such principles in a metaphysical inquiry. At least he seems to have held such a view with respect to his theory of types. Carnap, also, although he despised metaphysical speculation, and even though he protested with respect to other sorts of issues that such matters merely involve a choice of language framework, was apparently hornsoggled into believing that type theory was a true description of how things are in the universe.

6. Later in this paper I discuss the suggestion that the principle of bi-valence is at fault. At this particular point in the discussion I am not concerned to discuss or debate the pros and cons of mathematical intuitionism and three-valued logic but merely to distinguish these views from the view that any sentence which is cognitively meaningless is neither true or false.

7. Several issues and problems, although relevant to the matters discussed in the text, require more space than is available in this paper. Nevertheless, their brief mention is perhaps not out of order here. First of all, it is clear that belief language concerning deity can be understood metaphorically or analogically as well as literally. It would be appropriate therefore to show that the paradox is generated under a metaphorical or analogical interpretation of *belief* as well as a literal one. Until this has been done for the sentences in question, it may be objected that I have not clearly shown that God cannot believe all true sentences.

I am convinced that whether 'belief' is understood literally or not, the paradox in question is still generable. Reasoned support for this conviction is too extended to be included in this paper, and perhaps may be taken up at a later date.

Paul Tillich was an eminent advocate of the view that only non-literal language is truly applicable to deity. See his *Systematic Theology*, Univ. of Chi. Press, vol. 1, Chicago, Illinois, 1951, p. 239. "God is Being-itself or the absolute. However, after this has been said, nothing else can be said about God as God which is not symbolic." To be a symbolic assertion, for Tillich, is to be, among other things, non-literal. He seldom if ever speaks of metaphor or metaphorical language. This, I believe, is due to the fact that the metaphor-literal dichotomy makes a cut in the uses of expressions at a different point than does the symbol-sign dichotomy with which Tillich was concerned. As with the metaphor-literal distinction I doubt that treating religious language as symbolic will avoid the paradoxes under discussion. My reasons, unfortunately, cannot be elaborated in the present paper.

8. See his *Anatomy of Inquiry*, Alfred A. Knopf, New York, 1963, pp. 88-110. Another of Scheffler's papers, "An Inscriptional Approach to Indirect Quotation," *Analysis*, vol. 14, 1954, pp. 83-90, is also relevant to the issues discussed here.

9. An eternal sentence is characterized by Quine as one whose truth-value remains the same in every context. Such a sentence is constructed by eschewing all indexical expressions in favor of definite descriptions and then removing the definite descriptions according to the Quine-Russell method. Simple ambiguities along with any amphibolies are likewise eliminated.

10. A principle suggested by Hector Neri Castañeda in dealing with a similar problem surely fails here. This principle he states as follows:

If a sentence of the form 'X knows that a person Y knows that . . . ' formulates a true statement, then the person X knows the statement formulated by the clause filling the blank ' . . . '

While Castañeda is well aware of possible counterexamples to his principle, the ones he cites all have to do with problems of indicator expressions such as 'I' and 'he himself.' For the other cases he seems to suggest the detachment is legitimate. He says, "'Jones knows that Smith knows that $2+5=7$ ' does entail 'Jones knows that $2+5=7$.'" Is his principle suppose to hold, then, for all statements lacking indicator expressions? The paradoxes under discussion in this paper suggest otherwise. See his "Omniscience and Indexical Reference," *Journal of Philosophy*, vol. 64, no. 7, p. 207.

It should be clear at the outset that use of this principle will require that 'p' take as substituends only eternal sentences. Also, indirect discourse, as usual, gives headaches. Again caution is required in trying to nominalize a sentence and to use the resulting nominalization in place of 'that p.' Although I may know that you know where you parked your car, it doesn't follow that I know

where you parked your car. But where 'p' takes as substituent an eternal sentence, there is no immediately obvious reason why the transitivity principle should be inoperative for the "ordinary" range of cases. We shall see, however, that even making this distinction does nothing to circumvent the paradoxes.

11. For the religious believer whose religious posits include angels, this particular issue need not pose an irremediable problem, provided, of course that angels likewise have infinite capacities to comprehend paradoxical arguments.

12. See his *Counterfactuals*, Harvard Univ. Press, Cambridge, 1973.

13. The referee mentioned earlier objected that a possible person is not an actual person and hence given the problem at hand, God would not know the paradoxical conclusion even indirectly because there would be no second person whose knowledge could be appealed to. This is certainly true of several treatments of possible worlds semantics but is not true of every such treatment. David Lewis in particular holds that possible worlds are *real* in some not too *outré* sense of that much abused expression. I would suppose that this sense of the reality of possible worlds would apply, in Lewis' treatment, to the denizens of such worlds.

14. Since this paradox is of the form of a dilemma, the negation of this claim (i.e., it is not the case that God can create a stone so large He can't lift it) must likewise lead to the conclusion that God lacks omnipotence, if the argument is to be valid. But the logical structure of the negation does not imply the wanted conclusion. As C. Wade Savage has convincingly argued, to say that it is not the case that God can create a stone so large that He can't lift it, is not to say that there is a task that God cannot perform. (See, "The Paradox of the Stone," *The Philosophical Review*, 76 (1967), pp. 74-79. Reprinted in Baruch Brody, (ed.), *Readings in the Philosophy of Religion*, Englewood Cliffs, NJ, Prentice-Hall, 1974, pp. 345-49.) Moreover, this negation is perfectly consistent with the claim that God can create stones of any poundage and can lift stones of any poundage. Of course it is clear for finite beings that there are tasks of lifting and creating which are beyond our ability. The logical error in the case of divinity is to suppose that one can extrapolate from finite cases to infinite cases and to suppose further that since if we cannot do both A and B we are not omnipotent, then if God cannot do both A and B, he likewise is not omnipotent. Our inability to do something may constitute an inability on our part to perform a particular task. But God's inability (speaking very loosely) to create a stone so large that He can't lift it, is not an inability to perform a particular task. Failure to appreciate the differences between finite and infinite collections has frequently given rise to paradoxes of this sort. They are resolved by getting clear on the difference.

15. It is well known that this view is not universally accepted. Among the dissenters are Rene Descartes, Martin Luther, and John Calvin. These would each seem to hold that to say that God is unable to create a round square is to deny His omnipotence. The more reasonable view is that to claim that God cannot do what is logically impossible is to claim, not that there is a limit to what God can do, but rather that there is a limit to what constitutes an intelligible claim. It is a recognition of a limitation on our language rather than a recognition of a presumed limitation on the powers of deity that leads one to assert that God cannot do the impossible. Those who hold otherwise are not, as they suppose, upholding the divine mysteriousness of the creator but rather are simply showing that all such talk is utterly incoherent. I am not suggesting that such talk is cognitively meaningless. The difficulty attendant upon inconsistency is not too little meaning but too much. When I say that a claim is unintelligible what I mean is that it fails to fulfill the principal function of cognitive discourse: to distinguish what is the case from what is not the case. When I call a claim incoherent, I simply mean that there is no consistent interpretation of the sentence(s) making the claim (given

certain minimal assumptions concerning the use of the expressions of which the sentence is composed). To say, then, that the claim that God can create a square circle is incoherent and unintelligible is not to claim that it is cognitively meaningless. It is of course possible for a sentence or a string of expressions of a language to be unintelligible, incoherent, *and* cognitively meaningless. I am suggesting, however, that one ought not to *infer* cognitive meaninglessness from incoherence or unintelligibility.

I follow Quine in treating analytic sentences as those obtainable from logical truths by replacing synonym for synonym. Counteranalytic sentences are obtainable from logical falsehoods by replacing synonym for synonym. On this usage the set of counteranalytic sentences includes the set of logically impossible sentences.

16. The single quotes in the formula are to be read in this occurrence as Quinean corners with 'S' occurring as quantifiable variable.